Conic Sections

Question 1.
The straight line $y = mx + c$ cuts the circle $x^2 + y^2 = a^2$ in real points if
(a) $\sqrt{\{a^2 \times (1 + m^2)\}} < c$
(b) $\sqrt{\{a^2 \times (1 – m^2)\}} < c$
(c) $\sqrt{\{a^2 \times (1 + m^2)\}} > c$
(d) $\sqrt{\{a^2 \times (1 – m^2)\}} > c$

Answer: (c) $\sqrt{\{a^2 \times (1 + m^2)\}} > c$
The straight line $y = mx + c$ cuts the circle $x^2 + y^2 = a^2$ in real points if
$\sqrt{\{a^2 \times (1 + m^2)\}} > c$

Question 2.
Equation of the directrix of the parabola $x^2 = 4ay$ is
(a) $x = -a$
(b) $x = a$
(c) $y = -a$
(d) $y = a$

Answer: (c) $y = -a$
Given, parabola $x^2 = 4ay$
Now, its equation of directrix = $y = -a$

Question 3.
The equation of parabola with vertex at origin and directrix $x – 2 = 0$ is
(a) $y^2 = -4x$
(b) $y^2 = 4x$
(c) $y^2 = -8x$
(d) $y^2 = 8x$

Answer: (c) $y^2 = -8x$
Since the line passing through the focus and perpendicular to the directrix is x-axis, therefore axis of the required parabola is x-axis.
Let the coordinate of the focus is S(a, 0).

Since the vertex is the mid point of the line joining the focus and the point (2, 0) where the directix is x – 2 = 0 meets the axis.
So, 0 = {a – (-2)}/2
⇒ 0 = (a + 2)/2
⇒ a + 2 = 0
⇒ a = -2
Thus the coordinate of focus is (-2, 0)
Let P(x, y) be a point on the parabola.
Then by definition of parabola
$(x + 2)^2 + (y – 0)^2 = (x – 2)^2$
$\Rightarrow x^2 + 4 + 4x + y^2 = x^2 + 4 – 4x$
$\Rightarrow 4x + y^2 = – 4x$
$\Rightarrow y^2 = -4x – 4x$
$\Rightarrow y^2 = -8x$
This is the required equation of the parabola.

Question 4.
The perpendicular distance from the point (3, -4) to the line 3x – 4y + 10 = 0
(a) 7
(b) 8
(c) 9
(d) 10

Answer: (a) 7
The perpendicular distance = $\{3 \times 3 – 4 \times (-4) + 10\}/\sqrt{(3^2 + 4^2)}$
$= \{9 + 16 + 10\}/\sqrt{(9 + 16)}$
$= 35/\sqrt{25}$
$= 35/5$
$= 7$

Question 5.
The equation of a hyperbola with foci on the x-axis is
(a) $x^2/a^2 + y^2/b^2 = 1$
(b) $x^2/a^2 – y^2/b^2 = 1$
(c) $x^2 + y^2 = (a^2 + b^2)$
(d) $x^2 – y^2 = (a^2 + b^2)$

Answer: (b) $x^2/a^2 – y^2/b^2 = 1$
The equation of a hyperbola with foci on the x-axis is defined as
$x^2/a^2 – y^2/b^2 = 1$

Question 6.
If the line $2x – y + \lambda = 0$ is a diameter of the circle $x^2 + y^2 + 6x - 6y + 5 = 0$ then λ =
(a) 5
(b) 7
(c) 9
(d) 11

Answer: (c) 9
Given equation of the circle is
$x^2 + y^2 + 6x - 6y + 5 = 0$
Center O = (-3, 3)
radius $r = \sqrt{\{(-3)^2 + (3)^2 – 5\}} = \sqrt{\{9 + 9 – 5\}} = \sqrt{13}$
Since diameter of the circle passes through the center of the circle.
So (-3, 3) satisfies the equation $2x – y + \lambda = 0$
$\Rightarrow -3 \times 2 – 3 + \lambda = 0$
$\Rightarrow -6 – 3 + \lambda = 0$
$\Rightarrow -9 + \lambda = 0$
$\Rightarrow \lambda = 9$

Question 7.
The number of tangents that can be drawn from (1, 2) to $x^2 + y^2 = 5$ is
(a) 0
(b) 1
(c) 2
(d) More than 2

Answer: (b) 1
Given point (1, 2) and equation of circle is $x^2 + y^2 = 5$
Now, $x^2 + y^2 – 5 = 0$
Put (1, 2) in this equation, we get
$1^2 + 2^2 – 5 = 1 + 4 – 5 = 5 – 5 = 0$
So, the point (1, 2) lies on the circle.
Hence, only one tangent can be drawn.

Question 8.
The equation of the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ will represent a real circle if
(a) $g^2 + f^2 – c < 0$
(b) $g^2 + f^2 – c \geq 0$
(c) always
(d) None of these

Answer: (b) $g^2 + f^2 - c \geq 0$
Given, equation of the circle is: $x^2 + y^2 + 2gx + 2fy + c = 0$
This equation can be written as
$\{x - (-g)\}^2 + \{y - (-f)\}^2 + = \sqrt{\{g^2 + f^2 - c\}^2}$
So, the circle is real is $g^2 + f^2 - c \geq 0$

Question 9.
The equation of parabola whose focus is (3, 0) and directrix is $3x + 4y = 1$ is
(a) $16x^2 - 9y^2 - 24xy - 144x + 8y + 224 = 0$
(b) $16x^2 + 9y^2 - 24xy - 144x + 8y - 224 = 0$
(c) $16x^2 + 9y^2 - 24xy - 144x - 8y + 224 = 0$
(d) $16x^2 + 9y^2 - 24xy - 144x + 8y + 224 = 0$

Answer: (d) $16x^2 + 9y^2 - 24xy - 144x + 8y + 224 = 0$
Given focus S(3, 0)
and equation of directrix is: $3x + 4y = 1$
$\Rightarrow 3x + 4y - 1 = 0$
Let P (x, y) be any point on the required parabola and let PM be the length of the perpendicular from P on the directrix
Then, SP = PM
$\Rightarrow SP^2 = PM^2$
$\Rightarrow (x - 3)^2 + (y - 0)^2 = \{(3x + 4y - 1) / \{\sqrt{(3^2 + 4^2)}\}^2$
$\Rightarrow x^2 + 9 - 6x + y^2 = (9x^2 + 16y^2 + 1 + 24xy - 8y - 6x)/25$
$\Rightarrow 25(x^2 + 9 - 6x + y^2) = 9x^2 + 16y^2 + 1 + 24xy - 8y - 6x$
$\Rightarrow 25x^2 + 225 - 150x + 25y^2 = 9x^2 + 16y^2 + 1 + 24xy - 8y - 6x$
$\Rightarrow 25x^2 + 225 - 150x + 25y^2 - 9x^2 - 16y^2 - 1 - 24xy + 8y + 6x = 0$
$\Rightarrow 16x^2 + 9y^2 - 24xy - 144x + 8y + 224 = 0$
This is the required equation of parabola.

Question 10.
If the parabola $y^2 = 4ax$ passes through the point (3, 2), then the length of its latusrectum is
(a) 2/3
(b) 4/3
(c) 1/3
(d) 4

Answer: (b) 4/3
Since, the parabola $y^2 = 4ax$ passes through the point (3, 2)
$\Rightarrow 2^2 = 4a \times 3$
$\Rightarrow 4 = 12a$

$\Rightarrow a = 4/12$
$\Rightarrow a = 1/3$
So, the length of latusrectum = $4a = 4 \times (1/3) = 4/3$

Question 11.
The eccentricity of an ellipse is?
(a) $e = 1$
(b) $e < 1$
(c) $e > 1$
(d) $0 < e < 1$

Answer: (d) $0 < e < 1$
The eccentricity of an ellipse $e = \sqrt{(1 - a^2/b^2)}$ and $0 < e < 1$

Question 12.
If the length of the tangent from the origin to the circle centered at (2, 3) is 2 then the equation of the circle is
(a) $(x + 2)^2 + (y - 3)^2 = 3^2$
(b) $(x - 2)^2 + (y + 3)^2 = 3^2$
(c) $(x - 2)^2 + (y - 3)^2 = 3^2$
(d) $(x + 2)^2 + (y + 3)^2 = 3^2$

Answer: (c) $(x - 2)^2 + (y - 3)^2 = 3^2$
Radius of the circle = $\sqrt{\{(2 - 0)^2 + (3 - 0)^2 - 2^2\}}$
$= \sqrt{(4 + 9 - 4)}$
$= \sqrt{9}$
$= 3$
So, the equation of the circle = $(x - 2)^2 + (y - 3)^2 = 3^2$

Question 13.
If the length of the major axis of an ellipse is three times the length of the minor axis then its eccentricity is
(a) 1/3
(b) $1/\sqrt{3}$
(c) $1/\sqrt{2}$
(d) $2\sqrt{2}/\sqrt{3}$

Answer: (d) $2\sqrt{2}/\sqrt{3}$
Given, the length of the major axis of an ellipse is three times the length of the minor axis
$\Rightarrow 2a = 3(2b)$

$\Rightarrow 2a = 6b$
$\Rightarrow a = 3b$
$\Rightarrow a^2 = 9b^2$
$\Rightarrow a^2 = 9a^2 (1 - e^2)$ {since $b^2 = a^2(1 - e^2)$}
$\Rightarrow 1 = 9(1 - e^2)$
$\Rightarrow 1/9 = 1 - e^2$
$\Rightarrow e^2 = 1 - 1/9$
$\Rightarrow e^2 = 8/9$
$\Rightarrow e = \sqrt{(8/9)}$
$\Rightarrow e = 2\sqrt{2}/\sqrt{3}$
So, the eccentricity of the ellipse is $2\sqrt{2}/\sqrt{3}$

Question 14.
The equation of parabola with vertex at origin and directrix $x - 2 = 0$ is
(a) $y^2 = -4x$
(b) $y^2 = 4x$
(c) $y^2 = -8x$
(d) $y^2 = 8x$

Answer: (c) $y^2 = -8x$
Since the line passing through the focus and perpendicular to the directrix is x-axis, therefore axis of the required parabola is x-axis.
Let the coordinate of the focus is S(a, 0).
Since the vertex is the mid point of the line joining the focus and the point (2, 0) where the directix is $x - 2 = 0$ meets the axis.
So, $0 = \{a - (-2)\}/2$
$\Rightarrow 0 = (a + 2)/2$
$\Rightarrow a + 2 = 0$
$\Rightarrow a = -2$
Thus the coordinate of focus is (-2, 0)
Let P(x, y) be a point on the parabola.
Then by definition of parabola
$(x + 2)^2 + (y - 0)^2 = (x - 2)^2$
$\Rightarrow x^2 + 4 + 4x + y^2 = x^2 + 4 - 4x$
$\Rightarrow 4x + y^2 = - 4x$
$\Rightarrow y^2 = -4x - 4x$
$\Rightarrow y^2 = -8x$
This is the required equation of the parabola.

Question 15.
In an ellipse, the distance between its foci is 6 and its minor axis is 8 then its eccentricity is
(a) 4/5
(b) 1/√52
(c) 3/5
(d) 1/2

Answer: (c) 3/5
Given, distance between foci = 6
$\Rightarrow 2ae = 6$
$\Rightarrow ae = 3$
Again minor axis = 8
$\Rightarrow 2b = 8$
$\Rightarrow b = 4$
$\Rightarrow b^2 = 16$
$\Rightarrow a^2 (1 - e^2) = 16$
$\Rightarrow a^2 - a^2 e^2 = 16$
$\Rightarrow a^2 - (ae)^2 = 16$
$\Rightarrow a^2 - 3^2 = 16$
$\Rightarrow a^2 - 9 = 16$
$\Rightarrow a^2 = 9 + 16$
$\Rightarrow a^2 = 25$
$\Rightarrow a = 5$
Now, ae = 3
$\Rightarrow 5e = 3$
$\Rightarrow e = 3/5$
So, the eccentricity is 3/5

Question 16.
One of the diameters of the circle $x^2 + y^2 - 12x + 4y + 6 = 0$ is given by
(a) x + y = 0
(b) x + 3y = 0
(c) x = y
(d) 3x + 2y = 0

Answer: (b) x + 3y = 0
The coordinate of the centre of the circle $x^2 + y^2 - 12x + 4y + 6 = 0$ are (6, -2)
Clearly, the line x + 3y passes through this point.
Hence, x + 3y = 0 is a diameter of the given circle.

Question 17.
The center of the circle $4x^2 + 4y^2 – 8x + 12y – 25 = 0$ is?
(a) (2, -3)
(b) (-2, 3)
(c) (-4, 6)
(d) (4, -6)

Answer: (a) (2, -3)
Given, equation of the circle is $4x^2 + 4y^2 – 8x + 12y – 25 = 0$
$\Rightarrow x^2 + y^2 – 8x/4 + 12y/4 – 25/4 = 0$
$\Rightarrow x^2 + y^2 – 2x + 3y – 25/4 = 0$
Now, center = {-(-2), -3} = (2, -3)

Question 18.
If the parabola $y^2 = 4ax$ passes through the point (3, 2), then the length of its latusrectum is
(a) 2/3
(b) 4/3
(c) 1/3
(d) 4

Answer: (b) 4/3
Since, the parabola $y^2 = 4ax$ passes through the point (3, 2)
$\Rightarrow 2^2 = 4a \times 3$
$\Rightarrow 4 = 12a$
$\Rightarrow a = 4/12$
$\Rightarrow a = 1/3$
So, the length of latusrectum = $4a = 4 \times (1/3) = 4/3$

Question 19.
The equation of ellipse whose one focus is at (4, 0) and whose eccentricity is 4/5 is
(a) $x^2/5 + y^2/9 = 1$
(b) x2 /25 + y^2 /9 = 1
(c) $x^2/9 + y^2/5 = 1$
(d) $x^2/9 + y^2/25 = 1$

Answer: (b) x^2 /25 + y^2 /9 = 1
Given focus is (4, 0)
$\Rightarrow ae = 4$
and e = 4/5
$a \times (4/5) = 4$
$\Rightarrow a = 5$

Now, $b^2 = a^2 (1 – e^2)$
$\Rightarrow b^2 = 5^2 \{1 – (4/5)^2\}$
$\Rightarrow b^2 = 25\{1 – 16/25\}$
$\Rightarrow b^2 = 25\{(25 – 16)/25\}$
$\Rightarrow b^2 = 9$
Hence, the equation of the ellipse is $x^2/a^2 + y^2/b^2 = 1$
$\Rightarrow x^2/5^2 + y^2/9 = 1$
$\Rightarrow x^2/25 + y^2/9 = 1$

Question 20.
The focus of parabola $y^2 = 8x$ is
(a) (2, 0)
(b) (-2, 0)
(c) (0, 2)
(d) (0, -2)

Answer: (a) (2, 0)
Given, $y^2 = 8x$
General equation is $y^2 = 4ax$
Now, $4a = 8$
$\Rightarrow a = 2$
Now, focus = (a, 0) = (2, 0)
